CBCS SCHEME



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**17MAT41**

# Fourth Semester B.E. Degree Examination, July/August 2022
# Engineering Mathematics – IV

Time: 3 hrs. Max. Marks: 100

***Note: 1. Answer any FIVE full questions, choosing ONE full question from each module.***
***2. Use of statistical tables is permitted.***



## Module-1

**1** a. Use Taylors series method to find y(0.1) considering upto 4th degree term. If y(x) satisfies the equation $\frac{dy}{dx} = x - y^2$, $y(0) = 1$. **(06 Marks)**

b. Use fourth order Runge-Kutta method to find y(0.1) given that $\frac{dy}{dx} = 3e^x + 2y$, $y(0) = 0$ and $h = 0.1$. **(07 Marks)**

c. Apply Adams-Bashforth method to solve the equation $(y^2 + 1)dy - x^2dx = 0$ at x = 1 given y(0) = 1, y(0.25) = 1.0026, y(0.5) = 1.0206, y(0.75) = 1.0679. **(07 Marks)**

**OR**

**2** a. Use modified Euler's method to find y(0.1) given that $\frac{dy}{dx} = x^2 + y$, y(0) = 1 by taking h = 0.05 considering the accuracy upto two approximations in each step. **(06 Marks)**

b. Using Runge-Kutta method of fourth order solve $\frac{dy}{dx} + y = 2x$ at x = 1.1. Given that y = 3 at x = 1 initially. **(07 Marks)**

c. Apply Milne's method to find y(1.4) correct to four decimal places given $\frac{dy}{dx} = x^2 + \frac{y}{2}$ and following data y(1) = 2, y(1.1) = 2.2156, y(1.2) = 2.4649, y(1.3) = 2.7514. **(07 Marks)**

## Module-2

**3** a. Given the ordinary differential equation $y'' + xy' + y = 0$ and the following table of initial values. Find y(0.4) by applying Milne's method.

| x | 0 | 0.1 | 0.2 | 0.3 |
|---|---|---|---|---|
| y | 1 | 0.995 | 0.9801 | 0.956 |
| y′ | 0 | -0.0995 | -0.196 | -0.2867 |

**(06 Marks)**

b. If α and β are two distinct roots of $J_n(x) = 0$ then prove that $\int_0^1 x\, J_n(\alpha x) J_n(\beta x)\, dx = 0$. **(07 Marks)**

c. Express $x^3 + 2x^2 - 4x + 5$ in terms of Legendre polynomials. **(07 Marks)**

**OR**

4 a. Prove that $J_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \cos x$. **(06 Marks)**

b. By Runge-Kutta method, solve $y'' = x[y']^2 - y^2$ for x = 0.2 correct to four decimal places, using the initial conditions y = 1 and y′ = 0 when x = 0. **(07 Marks)**

c. Derive Rodrigue's formula $P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n}\left[(x^2-1)^n\right]$. **(07 Marks)**

## Module-3

5 a. Derive Cauchy-Riemann equations in polar form. **(06 Marks)**

b. Find the analytic function f(z) whose real part is $\frac{\sin 2x}{\cosh 2y - \cos 2x}$. **(07 Marks)**

c. Find the bilinear transformation which maps the points z = 1, i, -1 into w = 2, i, -2. **(07 Marks)**

**OR**

6 a. State and prove Cauchy's integral formula. **(06 Marks)**

b. Evaluate $\int_C \frac{(z^2+5)\,dz}{(z-2)(z-3)}$ using residue theorem $C: |z| = 4$. **(07 Marks)**

c. Discuss the transformation $w = e^z$. **(07 Marks)**

## Module-4

7 a. Derive mean and standard deviation of the binomial distribution. **(06 Marks)**

b. A communication channel receives independent pulses at the rate of 12 pulses per micro second. The probability of transmission error is 0.001 for each micro second. Compute the probabilities of i) no error during a micro second ii) one error per micro second iii) atleast one error per micro second iv) two errors v) atmost two errors. **(07 Marks)**

c. The joint probability distribution for two random variables X and Y is as follows.

| X \ Y | -3 | 2 | 4 |
|---|---|---|---|
| 1 | 0.1 | 0.2 | 0.2 |
| 3 | 0.3 | 0.1 | 0.1 |

Determine: i) Marginal distribution of X and Y ii) Covariance of X and Y iii) Correlation of X and Y. **(07 Marks)**

**OR**

8 a. Derive mean and standard deviation of exponential distribution. **(06 Marks)**

b. In a normal distribution 31% of the items are under 45 and 8% of the items are over 64. Find the mean and standard deviation of the distribution. **(07 Marks)**

c. A fair coin is tossed thrice. The random variables X and Y are defined as follows X = 0 or 1 according as head or tail occurs on the first toss, Y = Number of heads.

i) Determine the distributions of X and Y.
ii) Determine the joint distributions of X and Y.
iii) Obtain the expectations of X and Y. **(07 Marks)**

## Module-5

**9** a. The weights of 1500 ball bearings are normally distributed with a mean of 635 grams and standard deviation of 1.36 grams. If 300 random samples of size 36 are drawn from this population, determine the expected mean and standard deviation of the sampling distribution of means if sampling is done. i) With replacement ii) Without replacement. **(06 Marks)**

b. Two horses A and B were tested according to the time (in seconds) to run a particular race with the following results:

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Horse A: | 28 | 30 | 32 | 33 | 33 | 29 | 34 |
| Horse B: | 29 | 30 | 30 | 24 | 27 | 29 | |

Test whether you can discriminate between the two horses. **(07 Marks)**

c. Show that $P = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$ is a regular stochastic matrix. **(07 Marks)**

**OR**

**10** a. A die is thrown 9000 times and a throw of 3 or 4 was observed 3240 times. Show that the die cannot be regarded as an unbiased one. **(06 Marks)**

b. Find the unique fixed probability vector for the regular stochastic matrix.

$$A = \begin{bmatrix} 0 & \frac{2}{3} & \frac{1}{3} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$$ **(07 Marks)**

c. Three boys A, B, C are throwing ball to each other. A always throws the ball to B and B always throws the ball to C. C is just as likely to throw the ball to B as to A. If C was the first person to throw the ball find the probabilities that after three throws
i) A has the ball ii) B has the ball iii) C has the ball. **(07 Marks)**

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